Maximum Likelihood Thresholding Algorithm Based on Four-Parameter Gamma Distributions

Peter De Ford and Geovanni Martinez

University of Costa Rica
School of Electrical Engineering
Image Processing and Computer Vision
Research Laboratory (IPCV-LAB)

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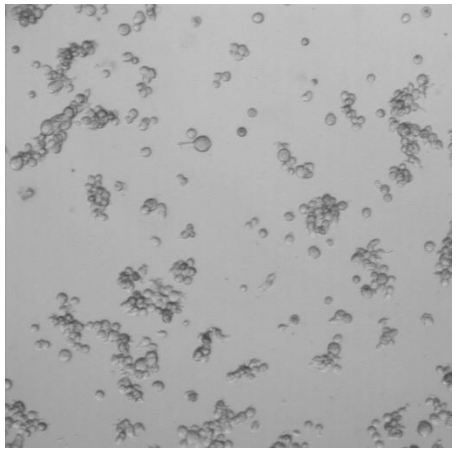
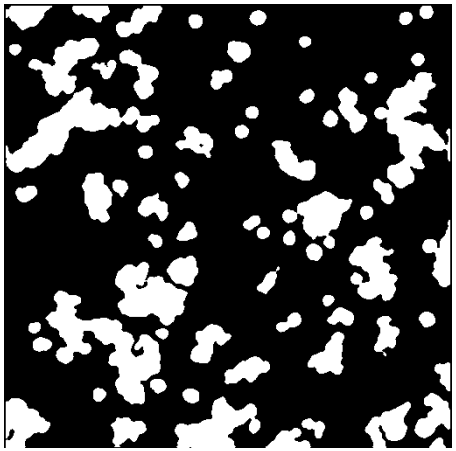
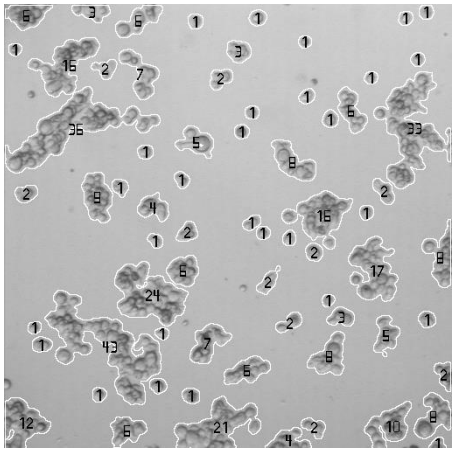
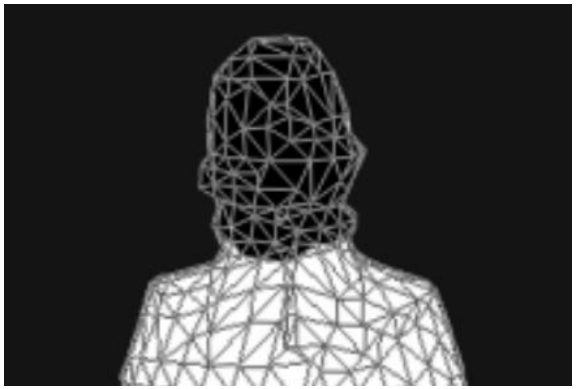
Introduction

IPCV-LAB research

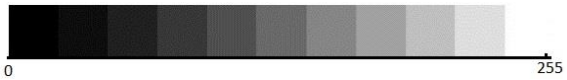


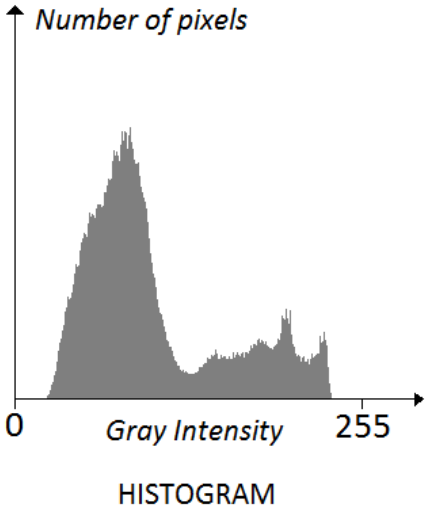


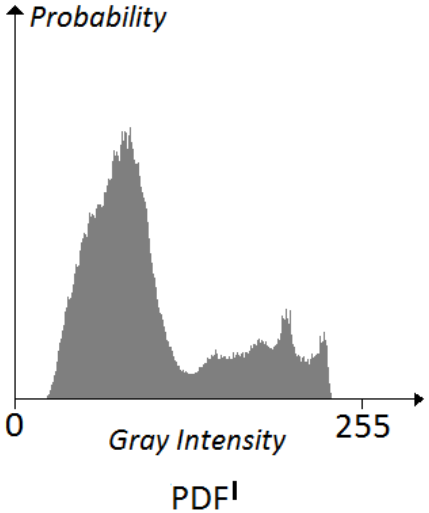


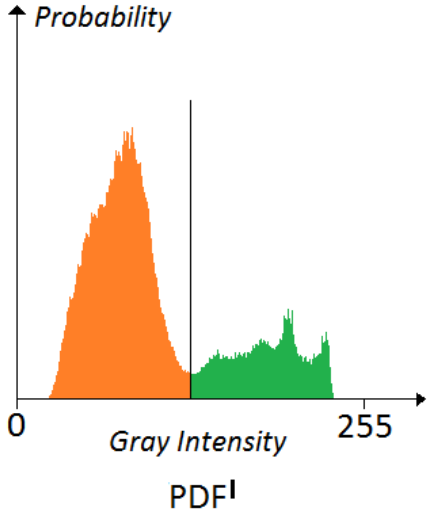


Thresholding algorithms





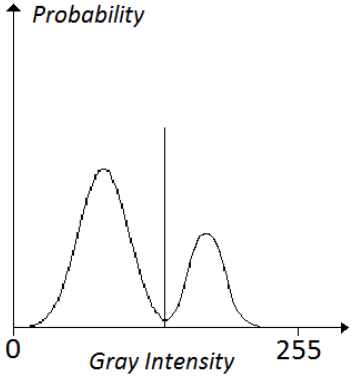




Maximum likelihood thresholding algorithms

Image formation model

Information Source

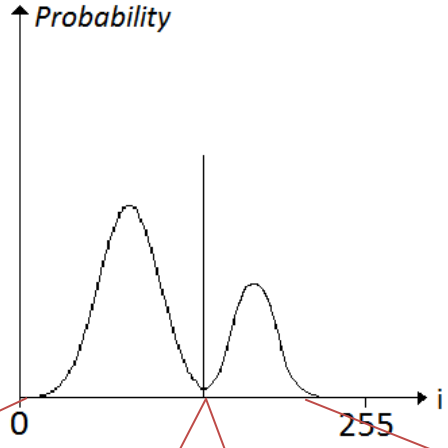


MODEL

112	115	118	117	112	111
108	123	109	134	148	115
109	112	131	185	183	170
111	105	151	189	190	183

Maximum likelihood thresholding algorithms

Information Source

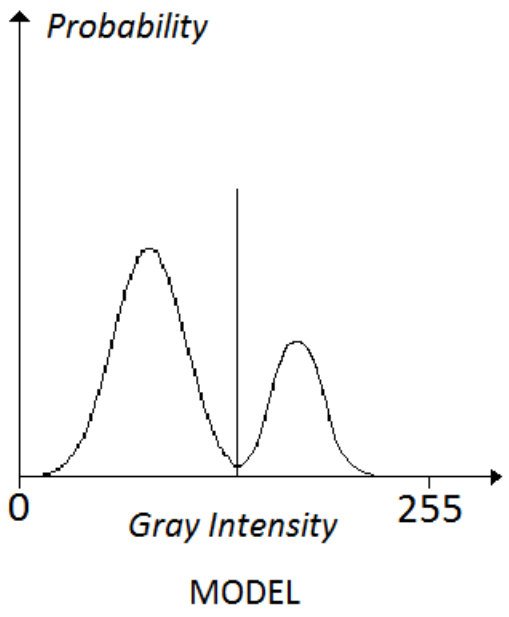


$$PDF^S(i;P) = w_0 * PDF_0^S(i;S_0) + w_1 * PDF_1^S(i;S_1)$$

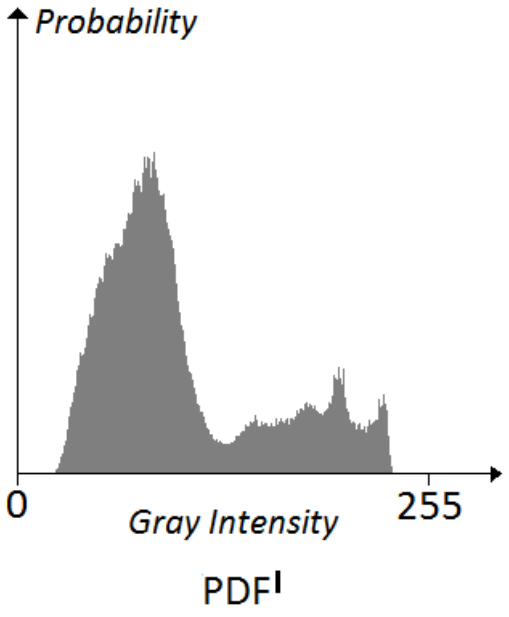
$$P: \{th, S_0, S_1, w_0, w_1\}$$

Maximum likelihood thresholding algorithms

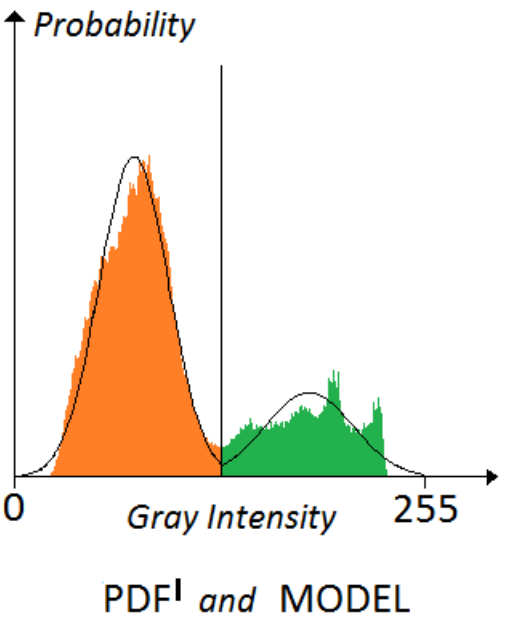
Optimum parameters



MODEL



PDF^{I}



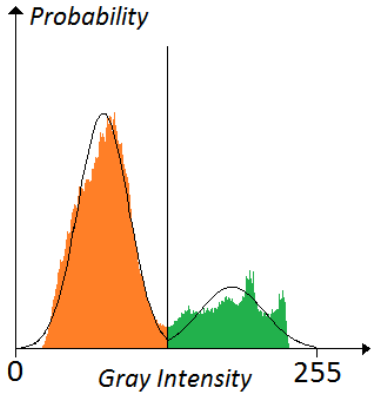
PDF^{I} *and* MODEL

$P: \{th, S_0, S_1, w_0, w_1\}$

Maximum likelihood thresholding algorithms

Likelihood function maximization

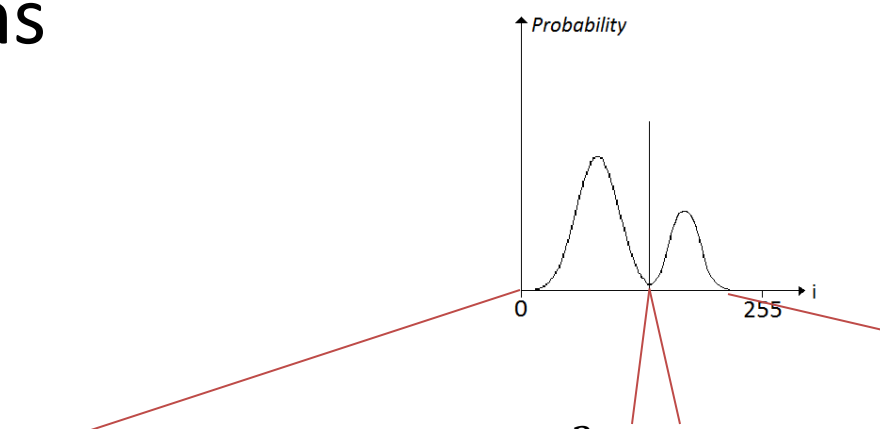
$$\ln\big(p(I|th)\big) = c_{0,th} * N * \ln(w_0) + c_{1,th} * N * \ln(w_1) + \sum_{i=0}^{th} h(i) \ln\Big(PDF_0^S(i;S_0)\Big) + \sum_{i=th+1}^{255} h(i) \ln\Big(PDF_1^S(i;S_1)\Big)$$



PDF and MODEL

Kittler, Kurita and Otsu's algorithms

Model based in two-parameter Gaussian distributions



$$PDF^S(i;P) = w_0 * \frac{1}{\sqrt{2\pi\sigma_0^2}} e^{\frac{-(i-\mu_0)^2}{2\sigma_0^2}} + w_1 * \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{\frac{-(i-\mu_1)^2}{2\sigma_1^2}}$$

$$P: \{th, \mu_0, \sigma_0^2, \mu_1, \sigma_1^2, w_0, w_1\}$$

Kittler, Kurita and Otsu's algorithms

Likelihood function

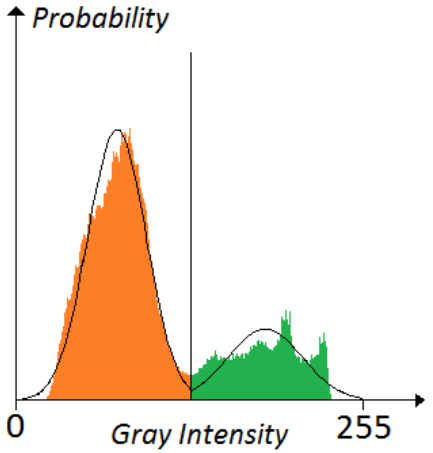
$$\ln\big(p(I|th)\big) = N\sum_{j=0}^{1} c_{j,th} * \ln\big(w_j\big) - \frac{N}{2} * \ln[2\pi] - \frac{N}{2}\sum_{j=1}^{2} c_{j,th}\ln\big[\sigma_j^2\big] - \frac{N}{2}$$

Optimum parameters estimation

$$P: \{th, \mu_0, \sigma_0^2, \mu_1, \sigma_1^2, w_0, w_1\}$$

$$th \rightarrow \{\mu_{0,th}, \sigma_{0,th}^2, \mu_{1,th}, \sigma_{1,th}^2, w_{0,th}, w_{1,th}\}$$

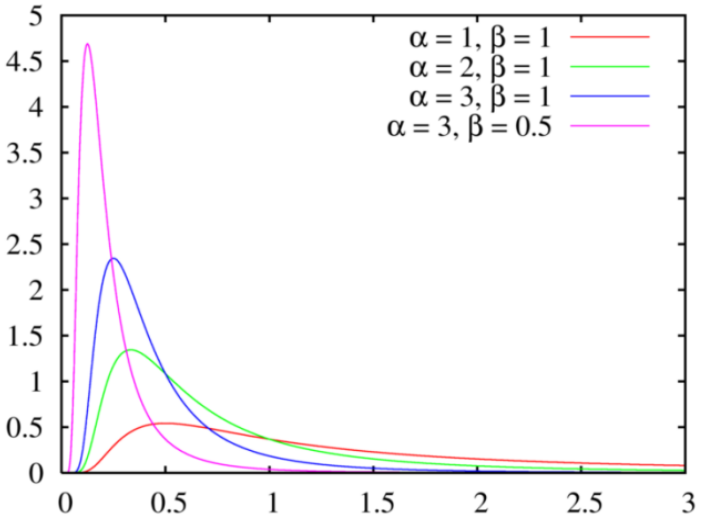
$$\{\hat{P}_0, \hat{P}_1, \dots, \hat{P}_{th}, \dots, \hat{P}_{254}, \hat{P}_{255}\}$$



PDF and MODEL

Proposed algorithm

Four-parameter gamma distribution

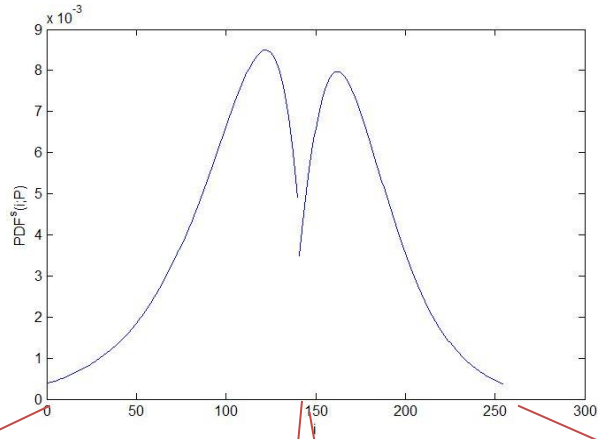


$$PDF\left(i;\underbrace{P}\right) = \frac{1}{\beta^{\alpha} * \Gamma(\alpha)}\left(\lambda(i-\varphi)\right)^{\alpha-1} e^{\frac{-\lambda(i-\varphi)}{\beta}}$$

$$P: \{\alpha, \beta, \varphi, \lambda\}$$

Proposed algorithm

Model based in four-parameter gamma distributions



$$PDF^{S}(i;P) = \frac{w_0 * (\lambda_0(i-\varphi_0))^{\alpha_0 - 1}}{{\beta_0}^{\alpha_0} * \Gamma(\alpha_0)} e^{\frac{-\lambda_0(i-\varphi_0)}{\beta_0}} + \frac{w_1 * (\lambda_1(i-\varphi_1))^{\alpha_1 - 1}}{{\beta_1}^{\alpha_1} * \Gamma(\alpha_1)} e^{\frac{-\lambda_1(i-\varphi_1)}{\beta_1}}$$

$$P: \{th, \alpha_0, \beta_0, \varphi_0, \lambda_0, \alpha_1, \beta_1, \varphi_1, \lambda_1, w_0, w_1\}$$

Proposed algorithm

Likelihoood function

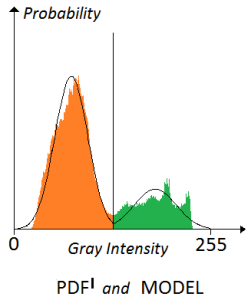
$$\ln\big(p(I|th)\big) = \{c_{0,th} * N * \ln(w_0) + c_{1,th} * N * \ln(w_1)\}$$
$$+\left\{-\ln({\beta_0}^{\alpha_0} * \Gamma(\alpha_0))\, c_{0,th} * N + \sum_{i=0}^{th}\left(h(i)\left((\alpha_0 - 1)\ln(\lambda_0(i-\varphi_0)) + \frac{-\lambda_0(i-\varphi_0)}{\beta_0}\right)\right)\right\}$$
$$+\left\{-\ln({\beta_1}^{\alpha_1} * \Gamma(\alpha_1))\, c_{1,th} * N + \sum_{i=th+1}^{255}\left(h(i)\left((\alpha_1 - 1)\ln(\lambda_1(i-\varphi_1)) + \frac{-\lambda_1(i-\varphi_1)}{\beta_1}\right)\right)\right\}$$

Optimum parameters estimation

$$P: \{th, \alpha_0, \beta_0, \varphi_0, \lambda_0, \alpha_1, \beta_1, \varphi_1, \lambda_1, w_0, w_1\}$$

$$th \rightarrow \{\alpha_{0,th}, \beta_{0,th}, \varphi_{0,th}, \lambda_{0,th}, \alpha_{1,th}, \beta_{1,th}, \varphi_{1,th}, \lambda_{1,th}, w_{0,th}, w_{1,th}\}$$

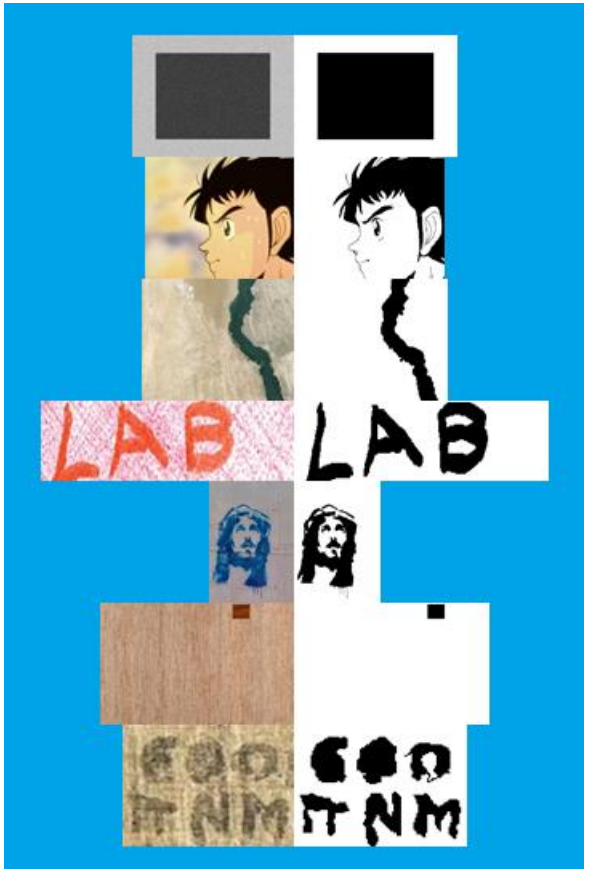
$$\{\hat{P}_0, \hat{P}_1, \dots, \hat{P}_{th}, \dots, \hat{P}_{254}, \hat{P}_{255}\}$$



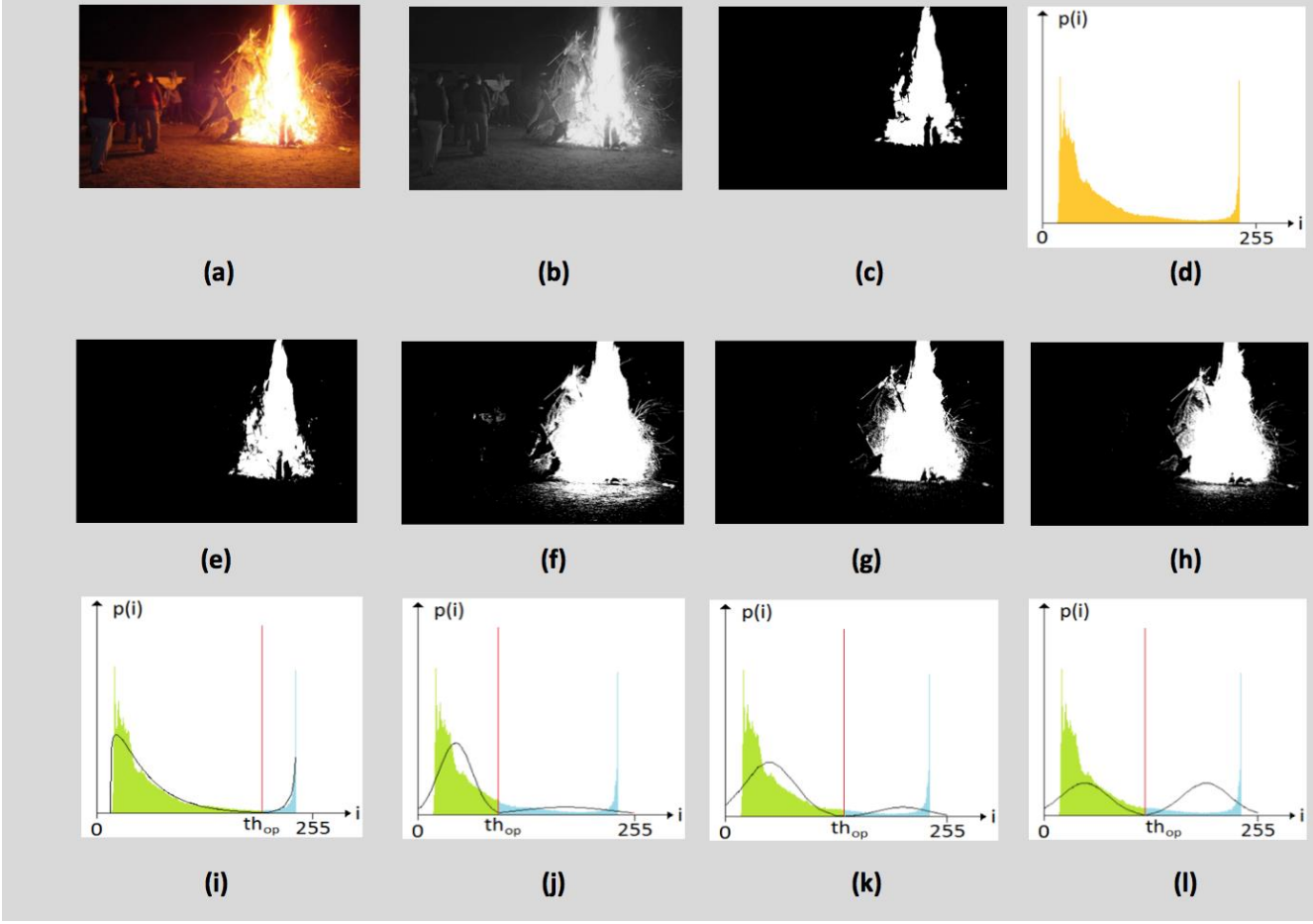
Evaluation

IPCV-LAB *Image Data Base*

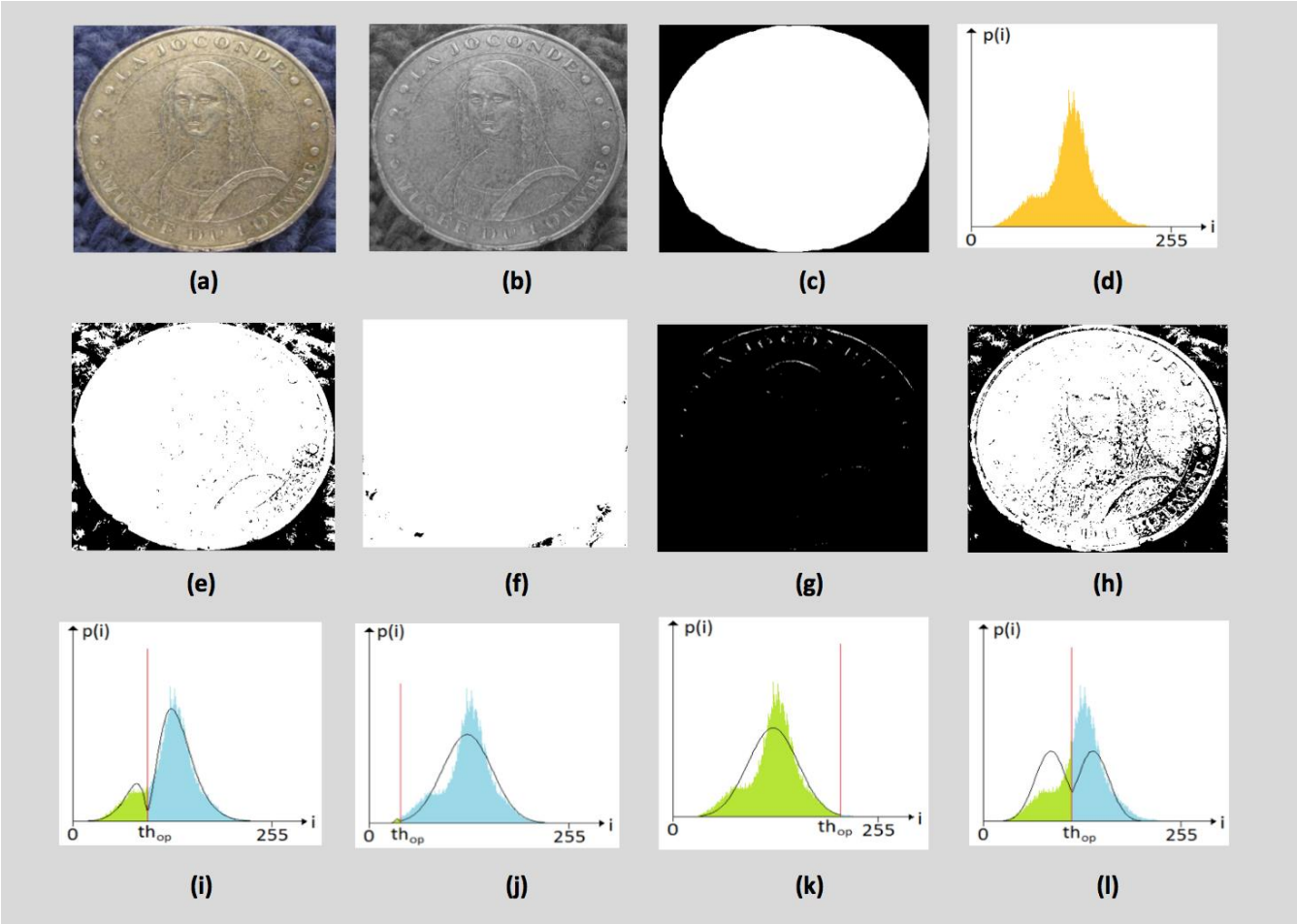
No.	Category	Images
1	Synthetic images	12
2	Computer graphics	8
3	Imaging	13
4	Letters	33
5	Photographs	42
6	Small objects	4
7	Low contrast	20



Example 1: Fire image



Example 2: Coin image



Evaluation: Mean Square Error

Category	Proposed	Kittler	Kurita	Otsu
Synthetic images	79	115	121	195
Computer graphics	301	350	687	894
Imaging	147	215	291	509
Letters	60	86	203	407
Photographs	58	86	150	271
Small objects	15	20	21	365
Low contrast	43	51	66	190
AVERAGE	**84**	**113**	**178**	**377**

*Numbers scaled by a 10^7 factor

Evaluation: F1-Score

Category	Proposed	Kittler	Kurita	Otsu
Synthetic images	0,947	0,950	0,984	0,974
Computer graphics	0,960	0,882	0,963	0,940
Imaging	0,952	0,892	0,965	0,960
Letters	0,922	0,912	0,914	0,936
Photographs	0,853	0,815	0,783	0,798
Small objects	0,999	0,997	0,999	0,670
Low contrast	0,785	0,631	0,614	0,690
AVERAGE	**0,92**	**0,87**	**0,89**	**0,85**

Summary and Conclusions

- In this contribution, we described clearly the theory behind maximum likelihood thresholding algorithms.
- We proposed a model based on four-parameter gamma distributions to describe the PDF^I, which intrinsically contains the models of Kittler, Kurita and Otsu's algorithms.
- The proposed algorithm approximated much better the PDF^I than the other three, and made much better segmentations for low contrast images and slightly better for general images.

Questions?

CONTACT INFORMATION
Peter De Ford and Geovanni Martínez, PhD.
Image Processing and Computer Vision Research Laboratory (IPCV-LAB)
Escuela de Ingeniería Eléctrica
Código postal 2060
Universidad de Costa Rica
San José, Costa Rica
Tel: +506 2511 3864
Fax: +506 2511 3920
E-mails: gmartin@eie.ucr.ac.cr and peterdeford@outlook.com
www: http://ipcv-lab.eie.ucr.ac.cr