Criterion for Automatic Selection of the Most Suitable Maximum-Likelihood Thresholding Algorithm for Extracting Object from their Background in a Still Image

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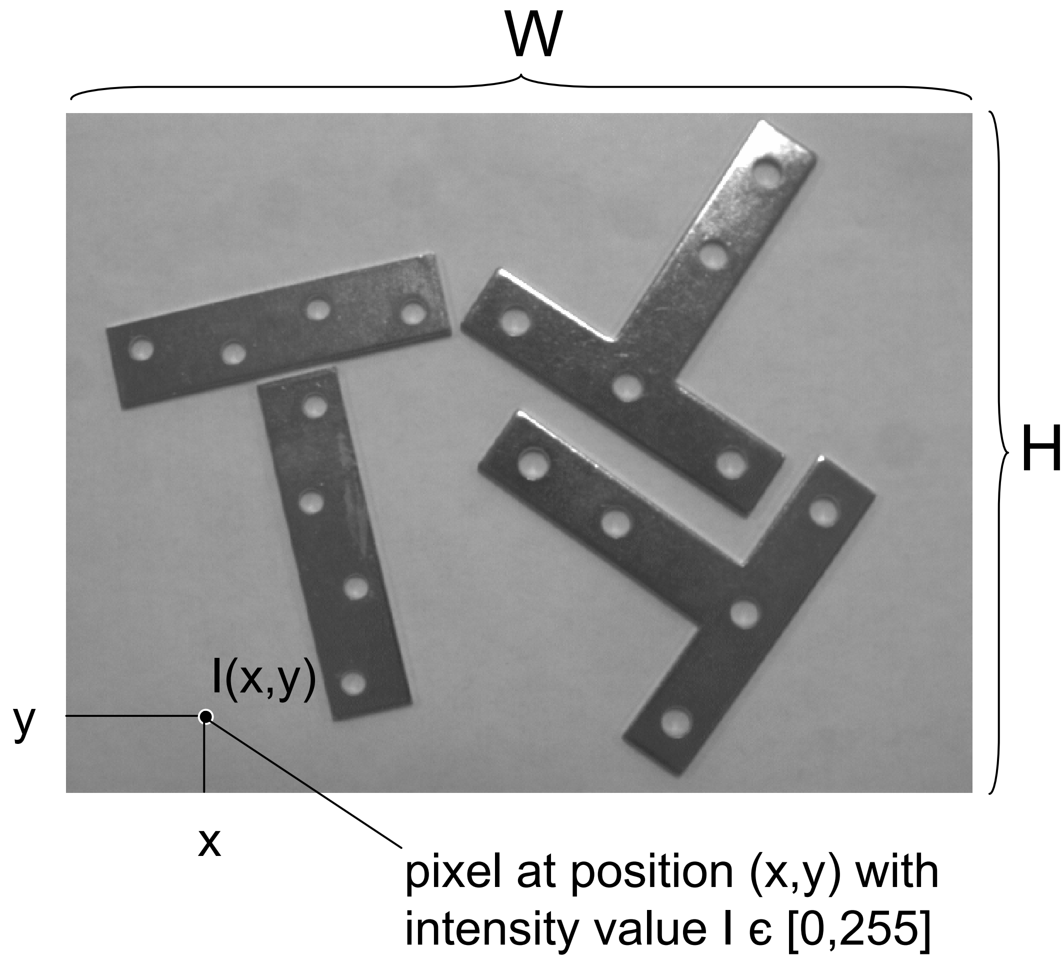
Intensity image

- Scene assumptions:
  - The objects intensity values are clearly different from background intensity values
  - The objects are darker than the background



Object segmentation

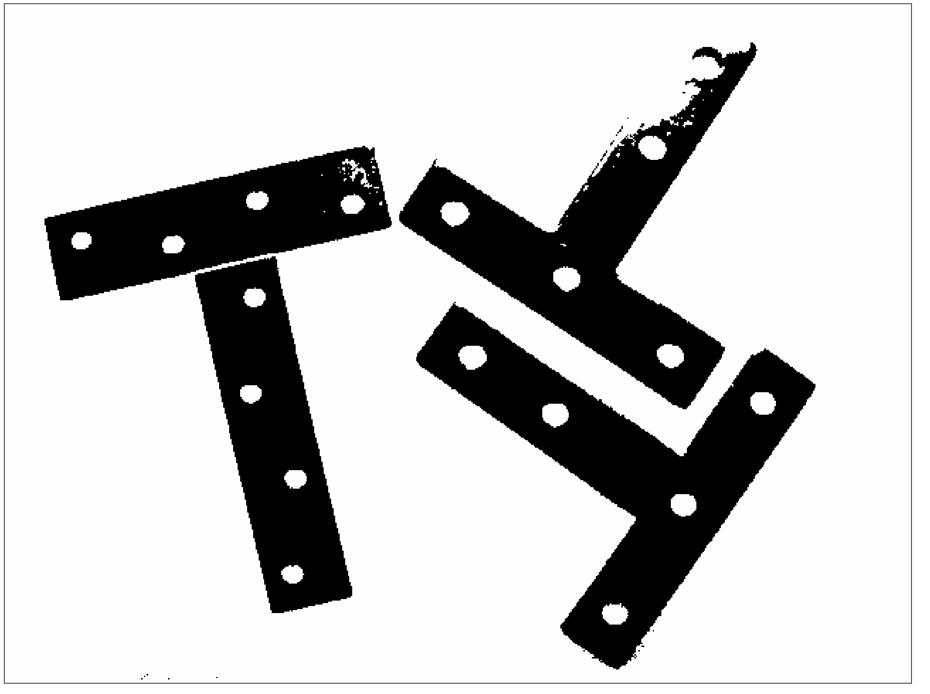
- The pixels are classified into two classes based on a comparison of their intensity values with the threshold k є [0,255]:

  if I(x,y)>k,

  I(x,y) є object

  if I(x,y)≤k,

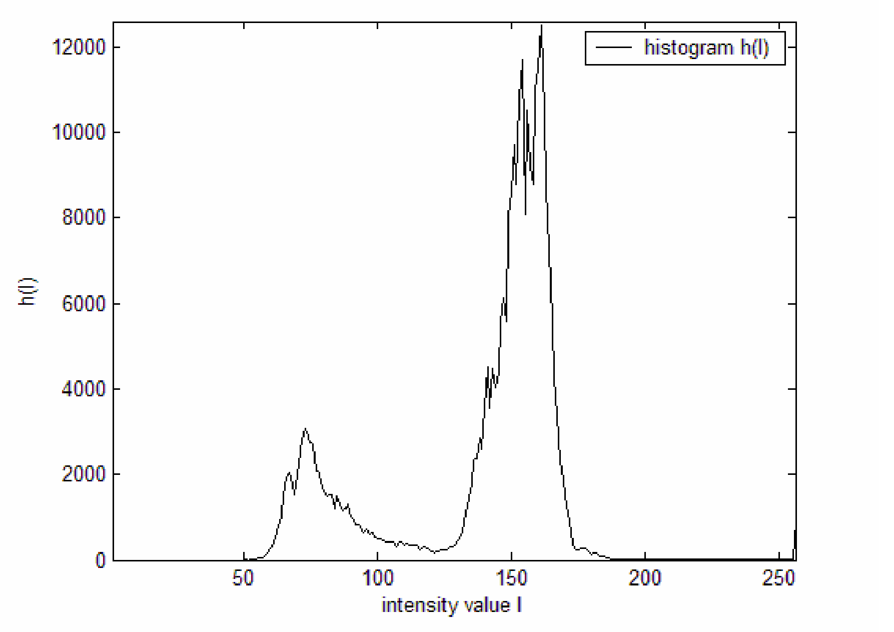
  I(x,y) є background



segmented image
k=127

Histogram

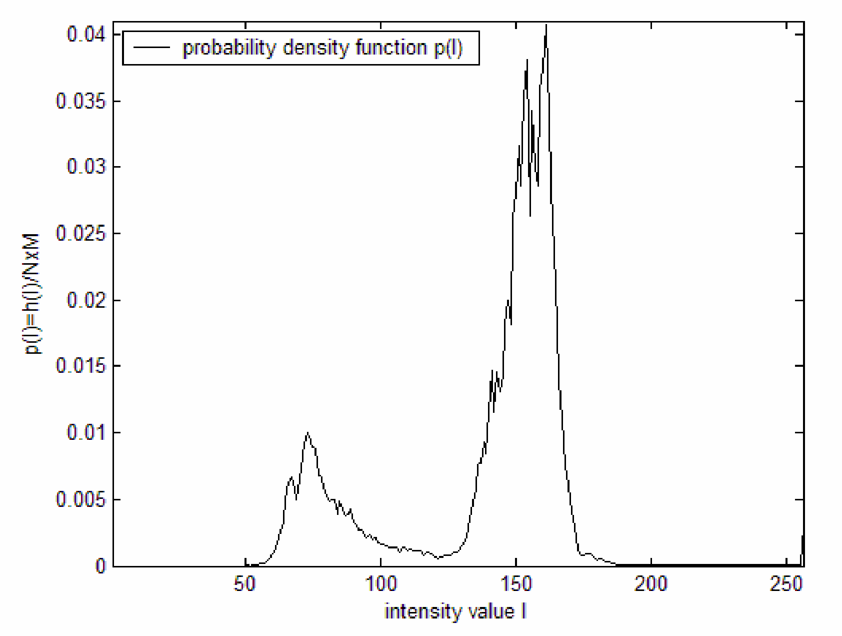
- The histogram h(I) gives the number of pixels in the image having the intensity value I



Probability Density Function

- The probability density function p(I) gives the probability that the intensity value I occurs in the image. It is estimated as the quotient between h(I) and the total number of pixels in the image:

$$p(I) = \frac{h(I)}{W \cdot H} = \frac{h(I)}{N}$$



Population Mixture Model

- The probability density function p(I) is described by the sum of two weighted Gaussian conditional probability functions:

$$p(I) = c_1 \cdot p_1(I/C_1) + c_2 \cdot p_2(I/C_2)$$

$$p(I) = \sum_{j=1}^{2} \frac{c_j}{\sqrt[2]{2\pi\sigma_j^2}} e^{-\frac{(I-m_j)^2}{2\sigma_j^2}}$$

$$c_1 + c_2 = 1$$

$$c_1(k) = \sum_{I=0}^{k} p(I)$$

$$c_2(k) = \sum_{I=k+1}^{255} p(I)$$

$$m_1(k) = \frac{1}{c_1(k)} \sum_{I=0}^{k} I \cdot p(I)$$

$$m_2(k) = \frac{1}{c_2(k)} \sum_{I=k+1}^{255} I \cdot p(I)$$

$$\sigma_1^2(k) = \frac{1}{c_1(k)} \sum_{I=0}^{k} (I - m_1)^2 \cdot p(I)$$

$$\sigma_2^2(k) = \frac{1}{c_2(k)} \sum_{I=k+1}^{255} (I - m_2)^2 \cdot p(I)$$

Optimal Threshold Estimation

- Otsu's algorithm
  - The optimal threshold k_1 is estimated by maximizing the likelihood function $L_1(k)$, which is the log of the conditional probability function of the pixels given their classes
  - $L_1(k)$ is computed describing p(I) by a population mixture model with different means and a common variance and assuming that the pixels are statistically independent

$$L_1(k_1) \geq L_1(k) \ \ \forall \ \ k = 0, 1, 2, ..., 255$$

$$L_1(k) = -\frac{N}{2} \cdot \log(2\pi) - \frac{N}{2} \log(\sigma_W^2(k)) - \frac{N}{2}$$

$$\sigma_W^2(k) = c_1(k) \cdot \sigma_1^2(k) + c_2(k) \cdot \sigma_2^2(k)$$

Optimal Threshold Estimation

- Kurita's algorithm
  - The optimal threshold k_2 is estimated by maximizing the likelihood function $L_2(k)$, which is the log of the joint probability function of the pixels and their classes
  - $L_2(k)$ is computed describing p(I) by a population mixture model with different means and a common variance and assuming that the pixels are statistically independent

$$L_2(k_2) \geq L_2(k) \ \forall \ k = 0, 1, 2, ..., 255$$

$$L_2(k) = N \cdot \sum_{j=1}^{2} c_j(k) \cdot \log(c_j(k)) + L_1(k)$$

Optimal Threshold Estimation

- Kittler's algorithm
  - The optimal threshold k_3 is estimated by maximizing the likelihood function $L_3(k)$, which is the log of the joint probability function of the pixels and their classes
  - $L_3(k)$ is computed by describing p(I) by a population mixture model with different means and different variances and assuming that the pixels are statistically independent

$$L_3(k_3) \geq L_3(k) \;\; \forall \;\; k = 0, 1, 2, ..., 255$$

$$L_3(k) = N \cdot \sum_{j=1}^{2} c_j(k) \cdot \log(c_j(k)) - \frac{N}{2} \log(2\pi) - \\ + \frac{N}{2} \cdot \sum_{j=1}^{2} c_j(k) \cdot \log(\sigma_j^2(k)) - \frac{N}{2}$$

Selection Criterion

1. Estimate the optimal thresholds k_1, k_2 and k_3 using the Otsu's algorithm, the Kurita's algorithm and the Kittler's algorithm, respectively
2. Estimate the population mixture model associated with each one of the previously estimated thresholds:

$$p_{k_1}(I) = \sum_{j=1}^{2} \frac{c_j(k_1)}{\sqrt[2]{2\pi\sigma_j^2(k_1)}} e^{-\frac{(I-m_j(k_1))^2}{2\sigma_j^2(k_1)}}$$

$$p_{k_2}(I) = \sum_{j=1}^{2} \frac{c_j(k_2)}{\sqrt[2]{2\pi\sigma_j^2(k_2)}} e^{-\frac{(I-m_j(k_2))^2}{2\sigma_j^2(k_2)}}$$

$$p_{k_3}(I) = \sum_{j=1}^{2} \frac{c_j(k_3)}{\sqrt[2]{2\pi\sigma_j^2(k_3)}} e^{-\frac{(I-m_j(k_3))^2}{2\sigma_j^2(k_3)}}$$

Selection Criterion

3. Compute the mean square error between the probability density function of the intensity values and each one of the previously estimated population mixture models:

$$mse(k_1) = \frac{1}{256}\sum_{I=0}^{255}\left[p_{k_1}(I) - p(I)\right]^2$$

$$mse(k_2) = \frac{1}{256}\sum_{I=0}^{255}\left[p_{k_2}(I) - p(I)\right]^2$$

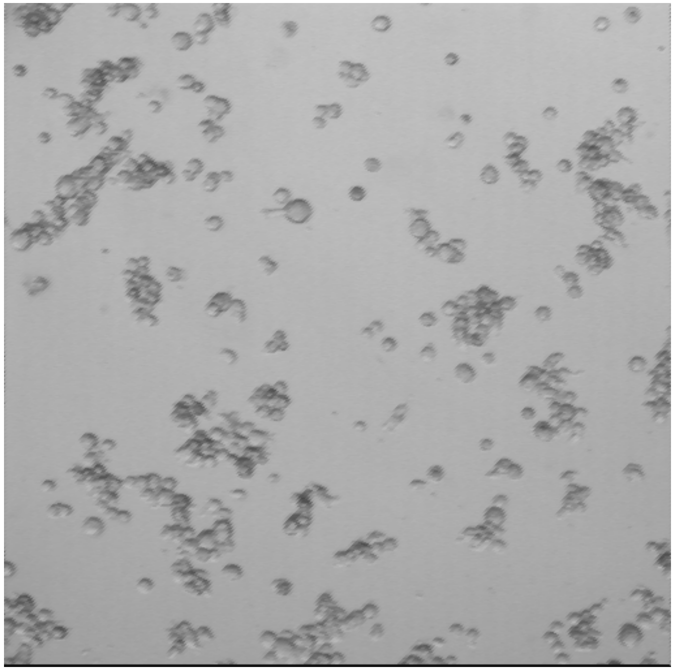
$$mse(k_3) = \frac{1}{256}\sum_{I=0}^{255}\left[p_{k_3}(I) - p(I)\right]^2$$

Selection Criterion

4. Select that algorithm whose estimated population mixture model produces the smallest mean square error:

$$mse(k_s) \leq mse(k), k = k_1, k_2, k_3$$

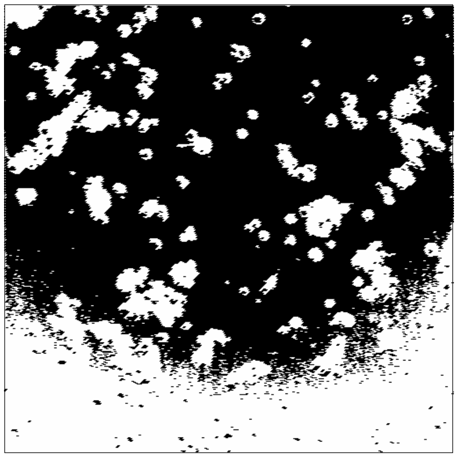
Results



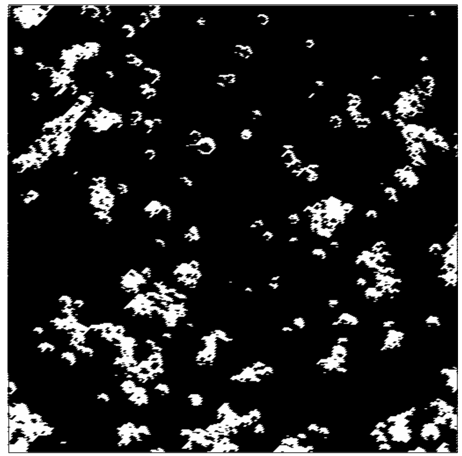
Baby Hamster Kidney cells

Dinosaur

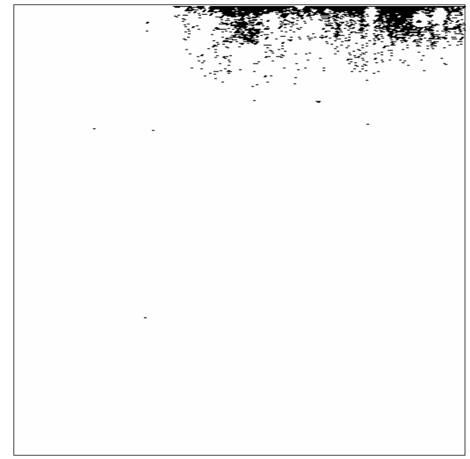
Results



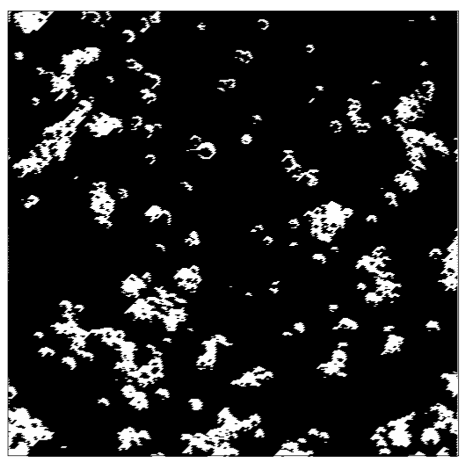
Otsu's algorithm



Kurita's algorithm

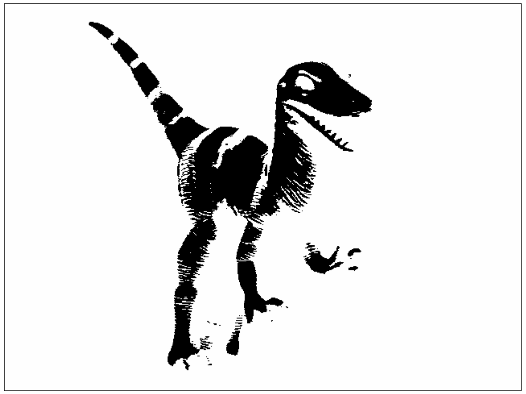


Kittler's algorithm

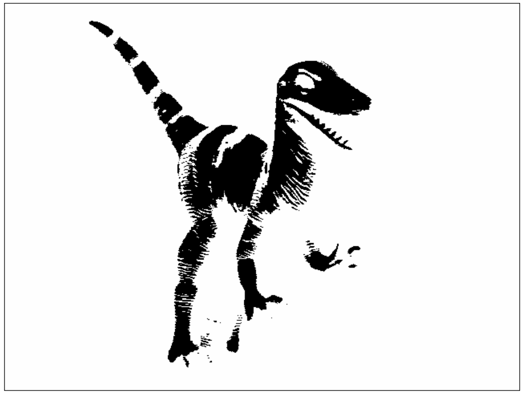


Kurita's algorithm (automatically selected)

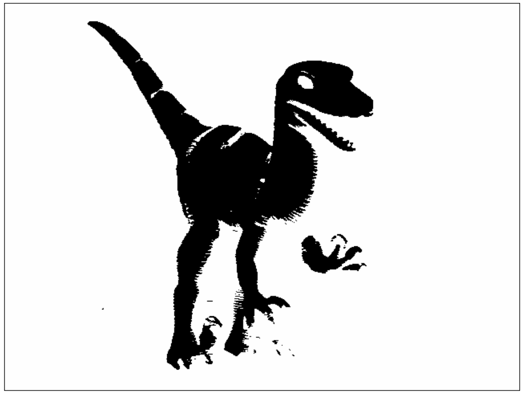
Results



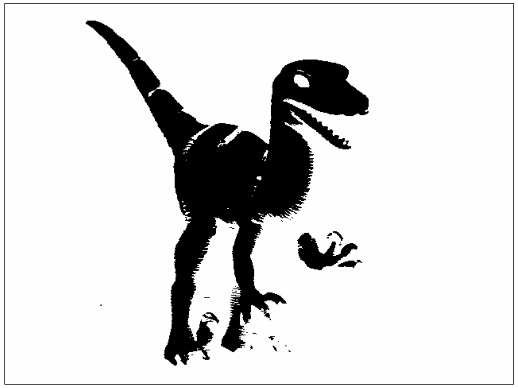
Otsu's algorithm



Kurita's algorithm



Kittler's algorithm



Kittler's algorithm (automatically selected)

Conclusions

- The selection criterion is based on a minimization of the mse between the probability density function of the intensity values and its approximation using a population mixture model

- A subjective analysis of the experimental results revealed that the proposed criterion was always able to select the algorithm which delivers the best thresholding results